Generating Special-purpose Stateless Propagators for Arbitrary Constraints

Ian Gent, Chris Jefferson,
Ian Miguel, Peter Nightingale

University
of St Andrews

Why is this the way forward?

- **59,049**

- **564,092,290**

How do you propagate an arbitrary new constraint?

- Hope it is one built in to your solver?
  - But it may not be
- Encode it using existing constraints?
  - Requires modelling time
  - May not achieve GAC
- Use table constraint implementation?
  - Can be exponential during search
  - Definitely not optimised to new constraint
- Write a new special purpose propagator?
  - At best takes programmer time
  - At worst is a significant research project

Instead, we show how to …

- Generate a special purpose propagator for an arbitrary new constraint
  - Either specified by table or existing propagator
- The generated propagator is …
  - created by code-generating code
  - guaranteed to achieve GAC every call
  - worst case time of $O(nd)$ per call during search
- Our best experimental results
  - Better than handwritten propagators
  - Achieved 18x speedup on "Oscillating Life"
    - Carrying out identical search to table constraint
  - Easily repaid overheads many times over

But …

- The overheads can be …
  - Exponential preprocessing before search
  - Exponential space needed during search
  - 10-ary boolean constraint close to our limit
- Overheads incurred per different table
  - So need constraint to occur many times
    - In a single problem
    - Or over many problems
  - Which does happen
    - Oscillating Life, compile constraint once, run often

DOMINION

- This work is part of the Dominion project
  - Generously funded by EPSRC
  - http://dominion.cs.st-andrews.ac.uk
  - Follow up to Minion
- *Generates* solver for every instance
  - i.e. is code-generating code
  - Produces a very special purpose solver!
  - Exploit individual features of instance
  - Make implementation choices for this instance
    - Which may not be good in a general purpose solver

Dominion

- For generating GAC propagators …
  - We therefore know domain size of all variables
  - And exact set of allowed tuples
  - *But* we will often see that propagators are reusable
- For technical reasons …
  - Propagators used in Minion, not Dominion
  - Technical reason being Dominion not existing

Basic Idea

- For *every* state domains of variables can be in
  - Compute what GAC does and store it
    - Either from propagator we are given …
    - … or from GAC-table propagator
- Generate code which
  - Looks up state of domains
  - Then does what the store tells it to

Basic Idea

- Constraint of arity *n*, domain size *d*
- There are 2^d-1 states of each variable
  - Excluding the empty domain
- So there are $(2^d-1)^n$ states of all variables
  - e.g. d = 2, n = 10 gives 59,049 states
- So it will pay to avoid considering all states
- Note that algorithm is *stateless*
  - i.e. it stores no state between invocations
  - All variants in this paper have this property

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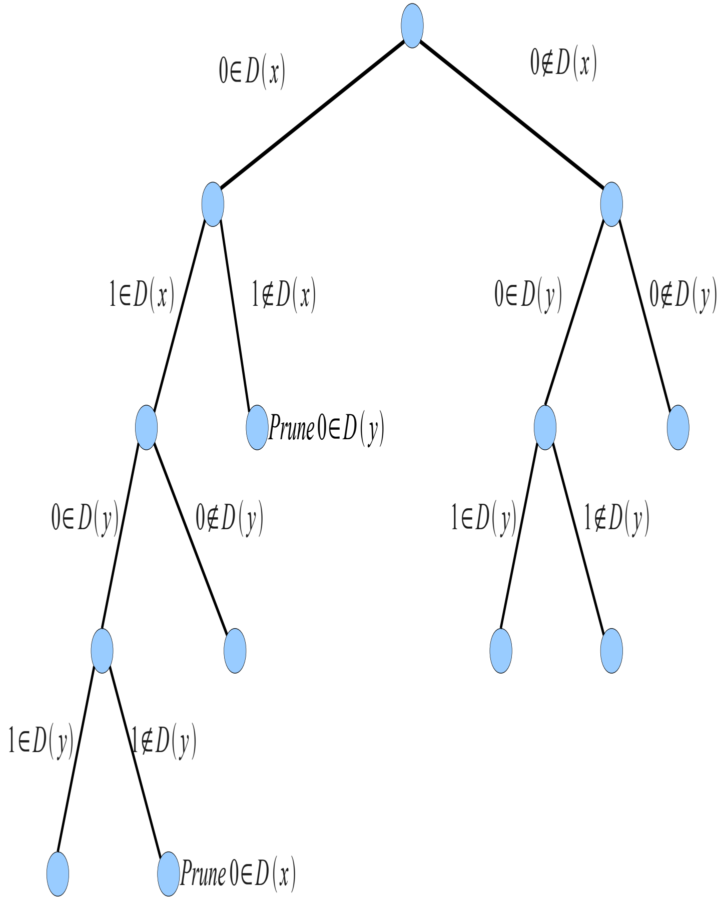
Second idea

- Build a binary decision tree
- Nodes store propagations
  - And next decision (unless leaf)
  - Leaves are sure to be GAC
- Decisions are domain membership
  - i in D(x)? for variable x value I
    - E.g. if domain of x is {1,3}
    - Then 1 in D(x), 2 not in D(x), 3 in D(x)
    - Even though 1 or 3 may later be removed
- Proved correct in this paper



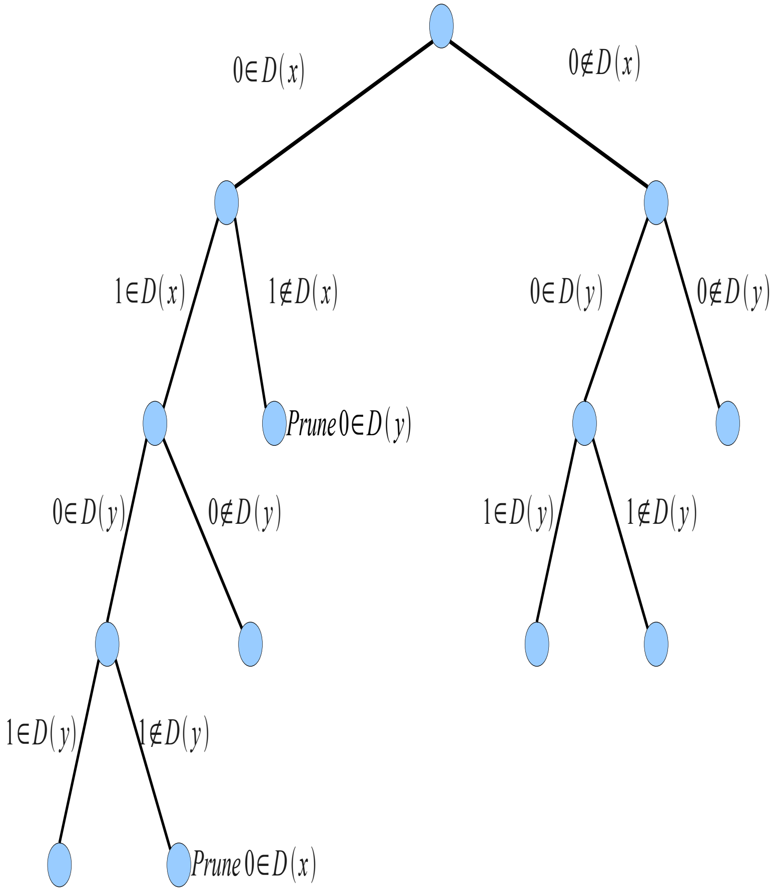
Tree for x v y, D(x) = D(y) = {0,1}

Binary decision tree

- There are only *nd* domain values
  - So max length of branch is *nd,* as is max run time
- GAC now to be run at each node in this tree
  - Instead of for every possible state of domains
  - If there are T nodes in tree then cost is
  - $O(T\ n\ d^n)$
- T can be as low as 1 for any n, d
  - E.g. domain wipeout at root

Third Idea

- Some nodes are redundant
  - Nodes where constraint is entailed
  - Other subtrees where no propagation happens
- These nodes can be turned into leaf nodes
  - Reduces space usage
  - Algorithm in paper & implemented
  - Not proved but "obviously" ok



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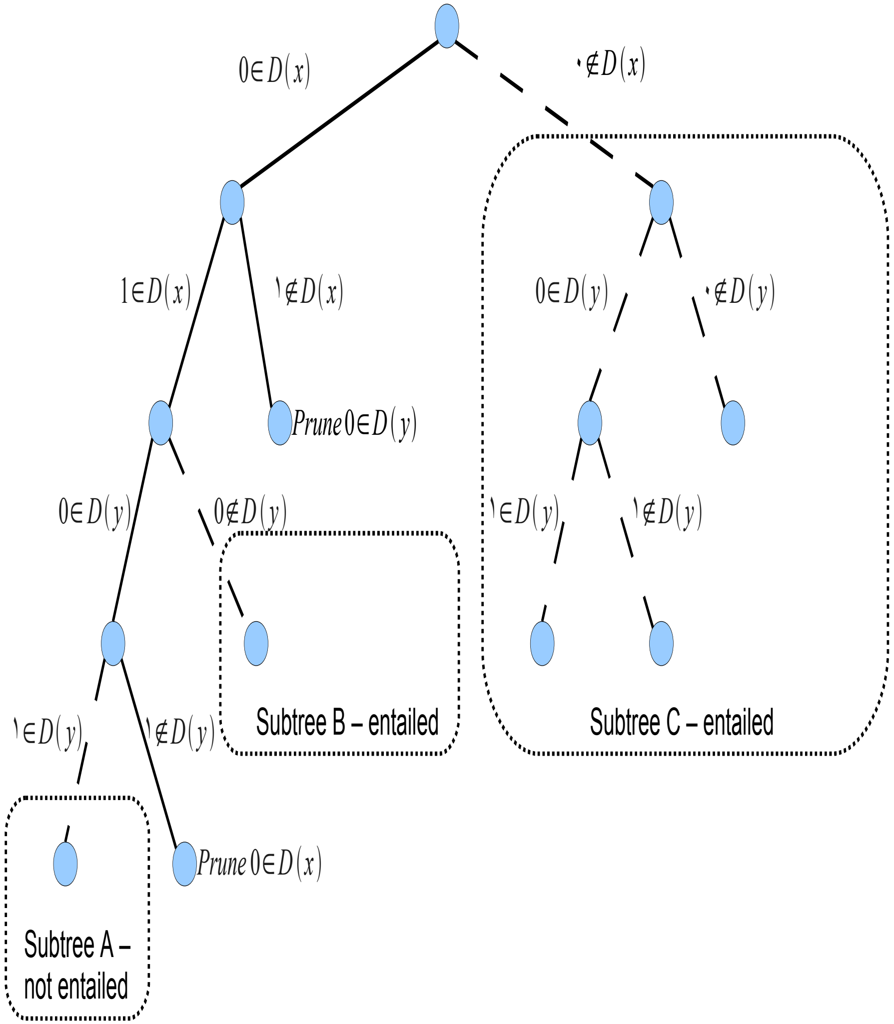

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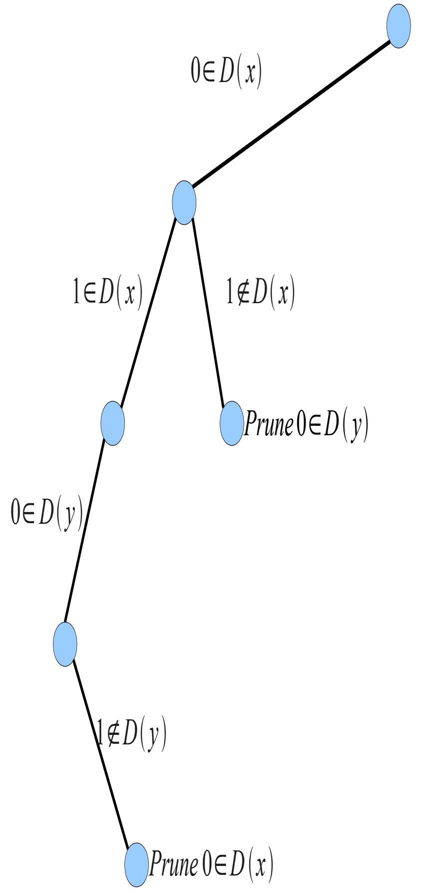


Heuristic

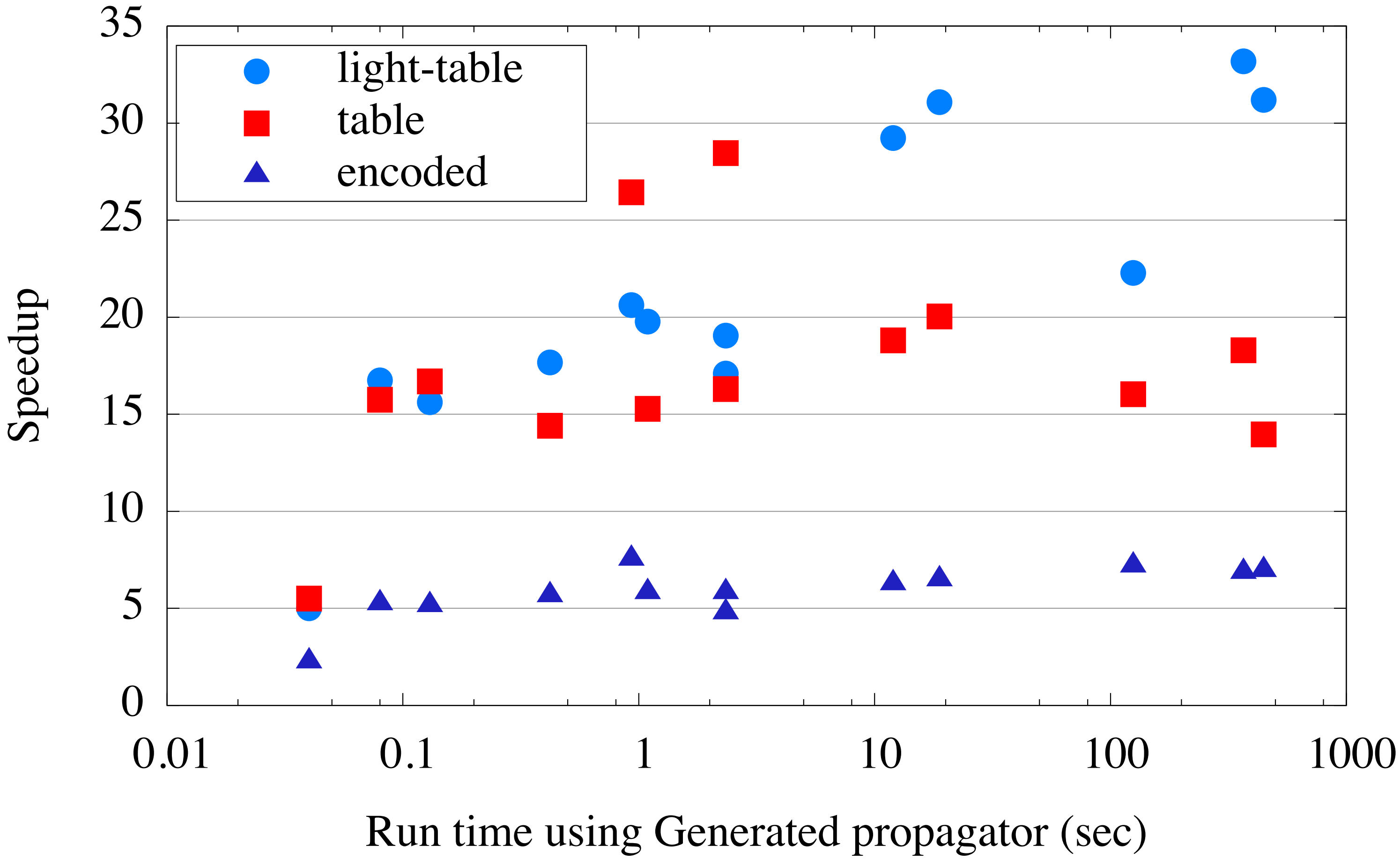
- Variable & value ordering
- Pick variable/value pair which
  - Is in maximum number of disallowed tuples
  - So that result is most likely to be entailed

Experiments

- Three case studies (great detail in paper)
  - Peg Solitaire
  - Low Autocorrelation Binary Sequences
  - Maximum Density Oscillating Still Life
- In each case constraint can be reused
  - So overheads can be amortised over all instances
- Compare against various other techniques
  - Minion GAC-table (using tries)
  - Minion "light table"
  - Encodings using other constraints

Oscillating Life Results

(overhead = 262s)



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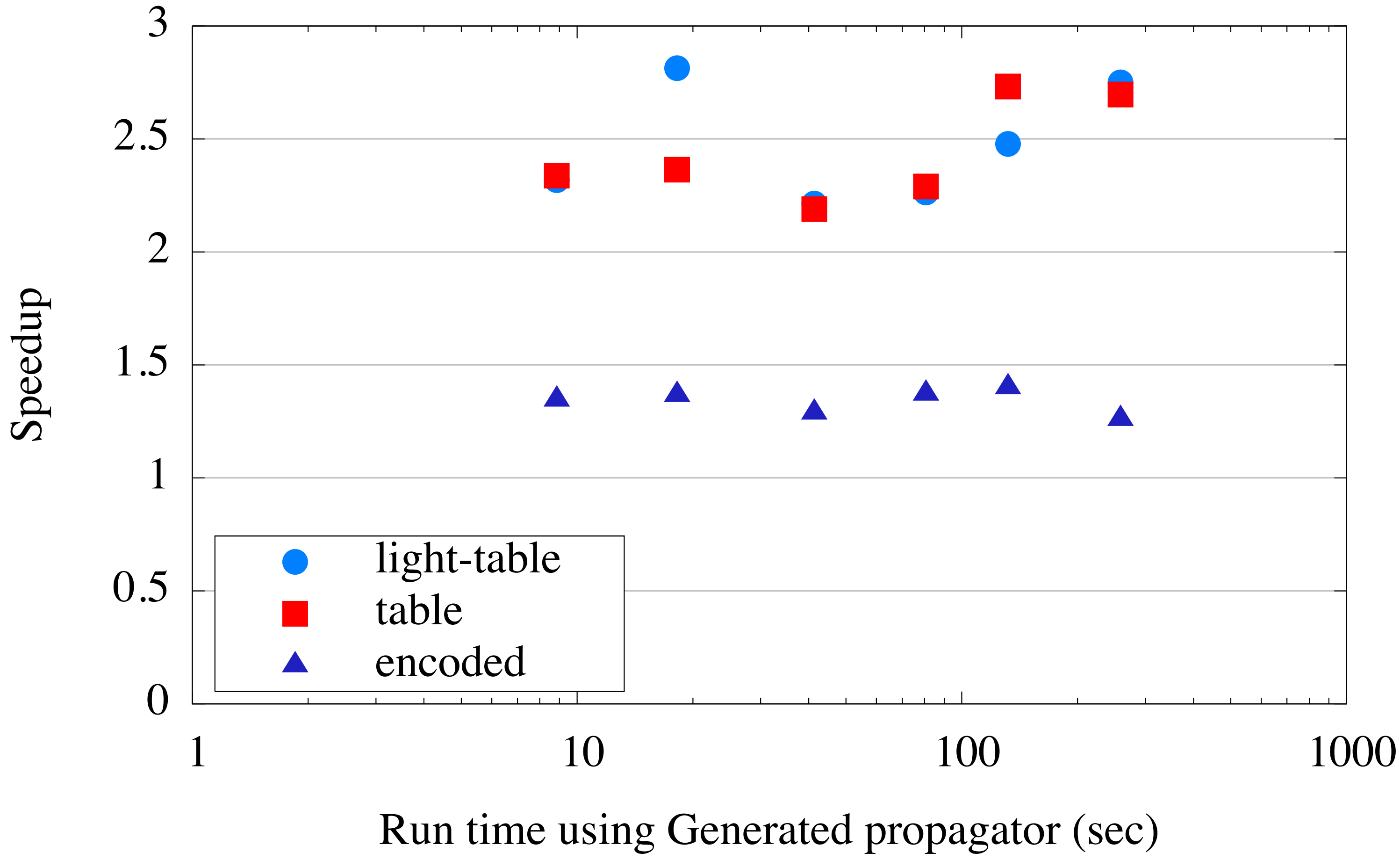
  - nodes in just one search

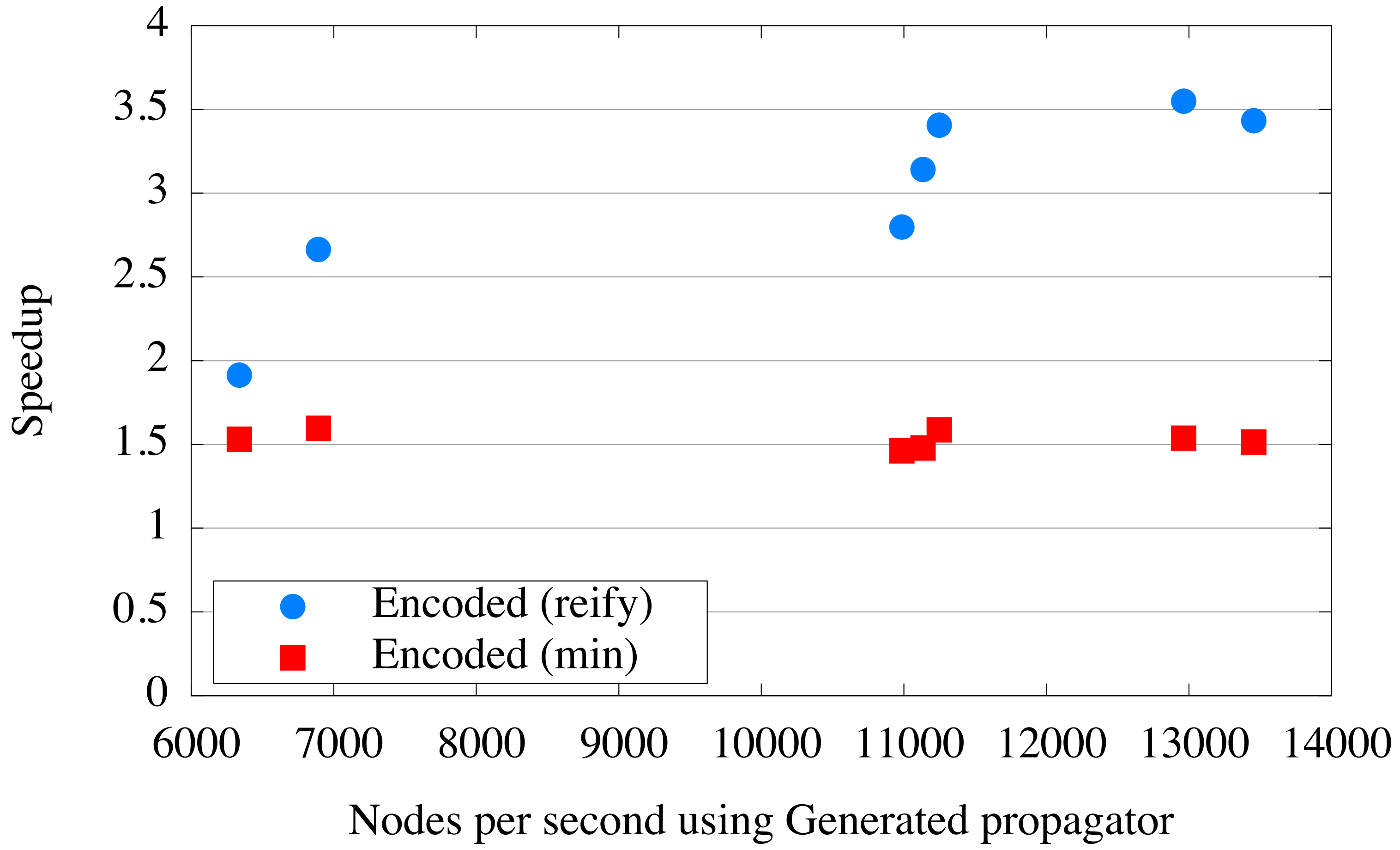
LABS Results

(overhead =15.7s)



Peg Solitaire Results

(overhead = 16.6s)



Experiments: Key Points

- Generated *always* the fastest method
- Generated *much* faster than table/light table
  - Searching identical search space
- Generated repays overhead on hard instances
  - *And* can be reused across all instances in family
- Still faster than encodings
  - Using hand-optimised propagators!
  - And encodings need thought

Relationship to other work

- Two main bodies of related work
  - Improving GAC for table constraint
    - Lhomme/Regin 05
    - Lecoutre/Szymanek 06,
    - Cheng/Yap 06 and 10,
    - Gent/Jefferson/Miguel/Nightingale 07,
    - Katsirelos/Walsh 07
    - Lecoutre/Hemery 07
  - Constructing sets of rules in CHR to establish GAC
    - Apt/Monfroy 01
- But no work (we believe) which generates GAC propagators to be polynomial during search

Conclusions

- We aim to push GAC propagators in new direction
    - Exponential preprocessing / poly during search
    - Dominion is perfect context as we have the full details of the problem before compilation
- Shown can obtain excellent results
- Many ways work can be extended

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If you have been …

… thanks for listening

Question

- Why not store the tree in a data structure instead of in code?
  - Avoids need for compiler at run time

- Yes, but compiled code runs even faster
- Even a 20% speedup can make the difference between winning and losing
- But something to consider certainly

Question

- What is the difference between entailed constraint and “no pruning”?

- Because of path to decision in binary tree
  - We *know* that some values are in domain

- E.g. x=/=y, original domains D(x) = D(y) = {0,1,2,3,4}
  - we might have asked
  - 0 in D(x)? Yes
  - 1 in D(x)? Yes
  - 0 in D(y)? Yes
  - 1 in D(y)? Yes

- It is now guaranteed that no propagation can occur
- But it is still possible that later we will set x = 0, and then propagate y =/= 0

Question

- Can you compute lazily?

- Yes …
  - Obvious to think of doing GAC when propagations not cached
  - I.e. memoize propagation
  - Gets rid of exponential preprocessing

- No …
  - Would need very different infrastructure to ours
  - Might increase cost of optimising data structure

- Obvious area of future work

Question

- Can you use BDDs/MDDs/Other data structure?
- No and Yes
- No …
  - not just by storing the constraint
  - We still need to do all propagations in advance
- Yes …
  - could compress data structure
  - i.e. which stores all the propagations

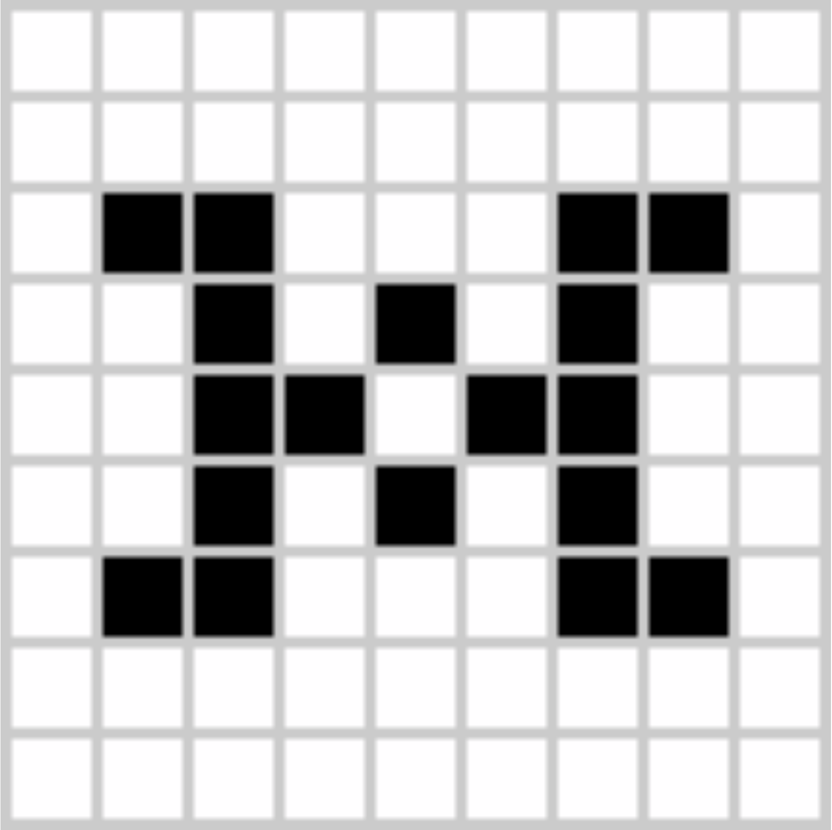
Question

- Is your run time really O(nd)?
- Is GAC-table really exponential during search?
- (Because data structure is exponential size)

- No…
  - True, input is much bigger so theoretical runtime is not what we said
- Yes…
  - After setup, exponential state is static and sits in RAM
  - In practice, access to data structure is O(1)

How to make oscillating life run 18 times faster

Minion table constraint



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